$U(1) \times SU(2)$ gauge theory of underdoped cuprate superconductors

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Abstract

The $U(1)\times SU(2)$ Chern-Simons gauge theory is applied to study the 2-D $t-J$ model describing the normal state of underdoped cuprate superconductors. The $U(1)$ field produces a flux phase for holons converting them into Dirac-like fermions, while the $SU(2)$ field, due to the coupling to holons gives rise to a gap for spinons. An effective low-energy action involving holons, spinons and a self-generated $U(1)$ gauge field is derived. The Fermi surface and electron spectral function obtained are consistent with photoemission experiments. The theory predicts a minimal gap proportional to doping concentration. It also explains anomalous transport properties including linear T dependence of the in-plane resistivity.

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The proximity of superconductivity (SC) to antiferromagnetism (AF) in reference compounds is a distinct feature of the high-T_c superconductors. Upon doping the AF goes away, giving rise to SC. At the same time, the Fermi surface (FS) is believed to develop from small pockets around $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$ [1], anticipated for a doped Mott insulator, to a large one around (π, π), expected from the electronic structure calculations. To understand this crossover is one of the key issues in resolving the high T_c puzzle. For this reason, the underdoped samples present particular interest due to the strong interplay of SC with AF. A "spin gap" or "pseudogap" has been invoked to explain the reduction of magnetic susceptibility χ below certain characteristic temperature T^* and suppression of the specific heat compared with the linear T behavior [2]. This gap also shows up in transport properties, neutron scattering, and NMR relaxation rate. The angle-resolved photoemission spectroscopy (ARPES) data show clear Fermi level crossing in the (0, 0) to (π, π) direction, but no such crossing was detected in the $(0, \pi)$ to (π, π) direction [3]. The observed pseudogap above T_c is consistent with d-wave symmetry. Theoretically there have been two competing approaches: One starting from the Mott-Hubbard insulator, advocated by Anderson [4] using the concept of spin liquid, while the other starting from the Fermi liquid (FL) point of view. Along the first line, P.A. Lee and his collaborators have consecutively developed the $U(1)$ [5] and $SU(2)$ [6] gauge theories, whereas the second approach has been elaborated by Kampt and Schrieffer [7], and Chubukov and his collaborators [8].

In this paper we apply to the $t-J$ model the $U(1) \times SU(2)$ Chern–Simons bosonization scheme for two-dimensional (2D) fermion systems [9]. This scheme provides a decomposition of the electron field into a product of two "semionic" fields, advocated by R.B. Laughlin [10], one carrying the charge (holon) and the other carrying the spin (spinon). It has been shown in [11] that a mean-field treatment of a dimensional reduction of such bosonization procedure, keeping the "semionic" nature of spinons and holons, reproduces the exact results obtained by Bethe–Ansatz and Luttinger liquid techniques, when applied to the 1D $t-J$ model at $t \gg J$. For the underdoped 2D $t-J$ model we neglect the feedback of holon fluctuations on the $U(1)$ field B and spinon fluctuations on the $SU(2)$ field V. The holon field is then a fermion and the spinon field a hard–core boson. Within this approximation we show that the B field produces a flux–phase for the holons, converting them into Dirac–like fermions; the V field, taking into account the feedback of holons produces a gap for the spinons vanishing in the zero doping limit, at $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$. A low–energy effective action in terms of spinons, holons and a self generated $U(1)$ gauge field is derived. Neglecting the gauge fluctuations, the holons are described by a FL with FS given by 4 "half-pockets" centered at $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$ and one reproduces the results for the electron spectral function obtained in mean field approximation (MFA) [13], in qualitative agreement with the ARPES data [3] for underdoped cuprates. Due to coupling to massless holons, gauge fluctuations are <u>not</u> confining, but nevertheless yield an attractive interaction between spinons and holons leading to a bound state in 2D with electron quantum numbers. The renormalisation effects due to gauge fluctuations induce non–FL behaviour for the composite electron, including the linear in T resistivity discussed earlier [14]. This formalism describes a smooth crossover upon doping from the long range ordered (LRO) AF state to short ranged (disordered) AF state with a gap in the excitation spectrum. The minimal gap is proportional to the doping concentration and the gap does not vanish in any direction.

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The euclidean action for the $t-J$ model in 2D can be represented in terms of a fermionic spinless holon field H, coupled to a $U(1)$ gauge field B, and a spin $\frac{1}{2}$ complex hard–core boson $\Sigma_\alpha, \alpha=1,2$, satisfying the constraint $\Sigma^*_\alpha\Sigma_\alpha=1$, coupled to an $SU(2)$ gauge field V, and it is given by [11]:

$$S=\int_0^\beta d\tau \sum_j [H^*_j\left(\partial_\tau - iB_0(j)-\delta\right)H_j + iB_0(j) + (1-H^*_jH_j)\Sigma^*_{j\alpha}\left(\partial_\tau + iV_0(j)\right)_{\alpha\beta}\Sigma_{j\beta}]$$

$$+\sum_{\langle ij\rangle}\Big[\Big(-tH^*_i e^{-i\int_{\langle ij\rangle}B}H_j\Sigma^*_{i\alpha}(Pe^{i\int_{\langle ij\rangle}V})_{\alpha\beta}\Sigma_{j\beta}+h.c.\Big)$$

$$+\tfrac{J}{2}(1-H^*_iH_i)(1-H^*_jH_j)(|\Sigma^*_{i\alpha}(Pe^{i\int_{\langle ij\rangle}V})_{\alpha\beta}\Sigma_{j\beta}|^2-\tfrac{1}{2})\Big]-2S_{c.s.}(B)+S_{c.s.}(V) \qquad (1)$$

where $P(\cdot)$ denotes the path–ordering, δ the chemical potential for the dopants and $S_{c.s.}(B)=\frac{1}{4\pi i}\int d^3x\epsilon_{\mu\nu\rho}B^\mu\partial^\nu B^\rho$, $S_{c.s.}(V)=\frac{1}{4\pi i}Tr\int d^3x\epsilon_{\mu\nu\rho}(V^\mu\partial^\nu V^\rho+\frac{2}{3}V^\mu V^\nu V^\rho)$ are the Chern–Simons action for the gauge fields (we refer to [11] for further details). The electron field at site j is decomposed as [9,11]: $c_{j\alpha}=e^{-i\int_{\gamma_j}B}H^*_j(Pe^{i\int_{\gamma_j}V})_{\alpha\beta}\Sigma_{j\beta}$, where γ_j is a straight line joining site j to ∞ in a fixed time plane.

The (local) gauge invariances of (1) are: $U(1): H_j\to H_je^{i\Lambda_j}, B_\mu(x)\to B_\mu(x)+\partial_\mu\Lambda(x),\ \Lambda(x)\in\mathbf{R}$; $SU(2):\Sigma_j\to R^\dagger_j\Sigma_j,\ V_\mu(x)\to R^\dagger(x)V_\mu(x)R(x)+R^\dagger(x)\partial_\mu R(x),\ R(x)\in SU(2)$; and holon/spinon (h/s) gauge: $H_j\to H_je^{i\xi_j},\ \Sigma_j\to e^{-i\xi_j}\Sigma_j,\ \xi_j\in\mathbf{R}$. We gauge–fix the $U(1)$ symmetry by imposing Coulomb condition for B. To retain the bipartite structure induced by AF interactions, we gauge–fix the $SU(2)$ symmetry by a "Nèel gauge" condition: $\Sigma_j=\sigma_x^{|j|}\binom{1}{0},\ |j|=j_1+j_2$. Now we split the integration over the V field into an integration over a field V^c satisfying the Coulomb condition, $\sum_{\mu=1}^2\partial^\mu V^c_\mu=0$ (from now on $\mu=1,2$), and its gauge transformations in terms of an $SU(2)$–valued scalar field g. Integrating over V_0 and B_0, we obtain

$$V^c_\mu=\sum_j(1-H^*_jH_j)(\sigma_x^{|j|}g_j^\dagger\tfrac{\sigma_a}{2}g_j\sigma_x^{|j|})_{11}\partial_\mu\text{arg }(x-j)\tfrac{\sigma_a}{2},$$

$$B_\mu=\bar{B}_\mu+\delta B_\mu,\quad \delta B_\mu(x)=\tfrac{1}{2}\sum_j H^*_jH_j\partial_\mu\text{arg }(x-j),$$

where $e^{i\int_{\partial p}\bar{B}}=-1$ for every plaquette p, and σ_a are the Pauli matrices.

Following the strategy in 1D [11], we write down the partition function of holons in a g background in terms of first quantized Feynman path integral, and find an *a priori* upper bound on it. We then look for a holon-dependent configuration $g^m, V^c(g^m)$ saturating the bound, taken as the starting point to add spinon fluctuations. This can be justified in the limit $t\gg J$, because the effective mass of holes is very heavy [15]. For an arbitrary given holon configuration the term $(\sigma_x^{|i|}g_i^\dagger P(e^{i\int_{\langle ij\rangle}V})g_j\sigma_x^{|j|})_{11}$ appears for a fixed link $\langle ij\rangle$ either in the "worldlines" of holons or in the Heisenberg term, but never simultaneously; this permits a separate optimization of the two cases (see [11]). Using techniques adapted from the proof of diamagnetic inequality [16], assuming translational invariance for the minimizing configuration g^m, neglecting the quartic pure holon term ($\delta\ll 1$) in (1), and making use of results of [17], it follows that for the optimal configuration (see [18] for further details): $V^c_\mu(g^m)=\bar{V}_\mu(g^m)+\sum_j\frac{(-1)^{|j|}}{2}\partial_\mu\text{arg }(x-j)\frac{\sigma_z}{2}$,

$$\bar V_\mu(g^m) = \sum_j (-1) H_j^* H_j \frac{(-1)^{|j|}}{2} \partial_\mu \arg\,(x-j)\frac{\sigma_z}{2} \tag{2}$$

and on links belonging to the holon worldlines

$$\left(\sigma_x^{|i|} g_i^{m\dagger} P(e^{i\int_{\langle ij\rangle} \bar V(g^m)}) g_j^m \sigma_x^{|j|}\right)_{11} \sim 1, \tag{3}$$

while on links in the Heisenberg term

$$\left(\sigma_x^{|i|} g_i^{m\dagger} P(e^{i\int_{\langle ij\rangle} \bar V(g^m)}) g_j^m \sigma_x^{|j|}\right)_{11} \sim 0. \tag{4}$$

Here $\bar V_\mu(g^m)$ is the slowly varying part of the $SU(2)$ gauge field related to holons, and the physical meaning of (3) and (4) will be explained later (after eq. (7)). We represent $g_j = \exp[-\frac{i}{2}\sum_{l\neq j}(-1)^{|l|}\sigma_z \arg(j-l)] R_j \exp[i\frac{\pi}{2}(-1)^{|j|}\sigma_y H_j^* H_j]$, where R is an $SU(2)$ – valued field, written in CP^1 form:

$$R_j = \begin{pmatrix} b_{j1} & -b_{j2}^* \\ b_{j2} & b_{j1}^* \end{pmatrix}, \quad b_{j\alpha}^* b_{j\alpha} = 1 \tag{5}$$

(no summation over j), describing the spinon fluctuations around g_j^m (for which $R_j = \hat I$). With suitable field redefinition, using the $SU(2)$ invariance of the g measure, the action of the $t-J$ model can be exactly rewritten as $S = S_h + S_s$, where

$$\begin{aligned} S_h &= \textstyle\int_0^\beta d\tau \sum_j H_j^* (\partial_\tau - (\sigma_x^{|j|} R_j^\dagger \partial_\tau R_j \sigma_x^{|j|})_{11} - \delta) H_j \\ &+ \sum_{\langle ij\rangle} [-t H_i^* e^{-i\int_{\langle ij\rangle} \bar B + \delta B} H_j (\sigma_x^{|i|} R_i^\dagger (P e^{i\int_{\langle ij\rangle} \bar V + \delta V}) R_j \sigma_x^{|i|})_{11} + h.c.] \\ S_s &= \textstyle\int_0^\beta d\tau \sum_j (\sigma_x^{|j|} R_j^\dagger \partial_\tau R_j \sigma_x^{|j|})_{11} \\ &+ \sum_{\langle ij\rangle} \tfrac{J}{2}(1-H_i^* H_i)(1-H_j^* H_j)\{|(\sigma_x^{|i|} R_i^\dagger (P e^{i\int_{\langle ij\rangle} \bar V + \delta V}) R_j \sigma_x^{|j|})_{11}|^2 - \tfrac{1}{2}\}. \end{aligned} \tag{6}$$

Notice that for small hole concentration ($\delta \ll 1$), $\bar V$ is a slowly varying field.

We now make the first approximation: suppose in (6) the fluctuations of the V field, due to the spinon fluctuations R are small enough that we can set $\delta V = 0$. Since the main effect of these fluctuations is to convert the $SU(2)$ gauge invariant spinon field into a semion field(see [9]), to be consistent, we neglect also the feedback of the holon field on B responsible for its semion nature, i.e. set $\delta B = 0$ as well. We believe that the proper account of the statistics of gauge–invariant spinon and holon fields is less crucial in 2D than in 1D, as we will see later. Let us consider the variable $R_i^\dagger P(^{i\int_{\langle ij\rangle} \bar V}) R_j =$

$$\begin{pmatrix} \alpha b_{i1}^* b_{j1} + \alpha^* b_{i2}^* b_{j2} & -\alpha b_{i1}^* b_{j2}^* + \alpha^* b_{i2}^* b_{j1}^* \\ -\alpha b_{i2} b_{j1} + \alpha^* b_{i1} b_{j2} & \alpha b_{i2} b_{j2}^* + \alpha^* b_{i1} b_{j1}^* \end{pmatrix}, \tag{7}$$

where $\alpha = \exp(\frac{i}{2}\int_{\langle ij\rangle} \bar V_z)$. In the hopping term of holons only the diagonal elements of (7) appear, a kind of gauge invariant Affleck-Marston (AM) variable [12]; in the Heisenberg

term only the off–diagonal elements appear, a kind of gauge invariant resonant valence bond (RVB) variable. According to the minimization arguments given above, the mean value of the AM gauge variable is s-like, real and close to 1 (see eq.(3)), while the RVB order parameter should be rather small (see eq.(4)). using a We now obtain a low–energy continuum action for spinons rescaling the model to a lattice spacing $\varepsilon \ll 1$ and neglecting higher order terms in ε. As it is standard in AF systems we define $\vec{n}_j = b^*_{j\alpha}\vec{\sigma}_{\alpha\beta}b_{j\beta}$ and assume [19]: $\vec{n}_j \sim \vec{\Omega}_j + (-1)^{|j|}\varepsilon\vec{L}_j,\ \vec{\Omega}_j^2 = f \leq 1$, (j)) + More precisely the fields $\vec{\Omega}, \vec{L}$ are defined on a sublattice, e.g. $\vec{\Omega}_j \equiv \vec{\Omega}_{j_1+\frac{1}{2},j_2}, \vec{L}_j \equiv \vec{L}_{j_1+\frac{1}{2},j_2}, j_1 = j_2 \bmod(2)$ and they describe the AF and ferromagnetic fluctuations, respectively. It is useful to write $\vec{\Omega}$ in CP^1 form: $\vec{\Omega} = z^*_\alpha\vec{\sigma}_{\alpha\beta}z_\beta$, with $z_\alpha, \alpha = 1,2$, a spin $\frac{1}{2}$ complex hard–core boson satisfying the constraint $z^*_\alpha z_\alpha = f$.

Evaluating the holon contribution in MFA, using the absence of the "θ" term in 2D [19], and integrating out over $\vec{L}$, we obtain a low–energy continuum limit NLσ model with action

$$S_s = \int d^3x \frac{1}{g}[(\partial_0\vec{\Omega})^2 + v_s^2(\partial_\mu\vec{\Omega})^2 + \vec{\Omega}^2\bar{V}_z^2] - \frac{1}{g}(\Omega_z)^2\bar{V}_z^2, \tag{8}$$

where g and v_s are easily derived in terms of $J, t, \delta, \varepsilon$.

To consider the effect of the $\bar{V}$ field, we replace the NLσ constraint $\vec{\Omega}^2 = f$ by a softened version adding to the lagrangian a term $\lambda(\vec{\Omega}^2 - f)^2$, and substitute $\bar{V}_z^2$ by its statistical averaging over holon configurations, $\langle\bar{V}_z^2\rangle$. Using a sine–Gordon transformation [20], we obtain $\langle\bar{V}_z^2\rangle \sim -\delta\ln\delta$ (see [18] for details). For small J, the coupling constant g is small and the system is in the ordered phase; the renormalization group flow in the absence of perturbation drives g_{eff} for large distances towards its critical value; the mass perturbation induced by $\langle\bar{V}_z^2\rangle$ should then drive the system from the ordered to the disordered phase (with short range order only). (One might speculate that if we treat the holons as slowly moving impurities, consistent with the known results in the limit $t \gg J$ [15], this would lead to a kind of "Anderson localization" considered in [21]). Hence our system should exhibit a mass gap $m(\delta)$ vanishing as $\delta \to 0$, and absence of AFLRO (at least for $\delta \ll 1$, but sufficiently big). This provides a smooth crossover to the insulating AF regime. The subleading perturbation appearing in eq.(8) gives rise to a remnant spin-space uniaxial AF interaction in short-ranged AF state.

We can summarize the above discussion by rewriting the NLσ model action in CP^1 form, neglecting the short range interactions, as

$$S_s^\star = \int d^3x \frac{1}{g}\left[|(\partial_0 - A_0)z_\alpha|^2 + v_s^2|(\partial_\mu + A_\mu)z_\alpha|^2 + m^2 z^*_\alpha z_\alpha\right] \tag{9}$$

In the NLσ model without mass term ($\delta = 0$), the constraint $z^*_\alpha z_\alpha = f$ and the symmetry breaking condition, e.g. $\langle z_1\rangle \neq 0$, lead to excitations described by a complex massless field $S \equiv \langle z_1^*\rangle z_2$, with relativistic massless dispersion relation corresponding to the spin waves. In the NLσ model with mass term the absence of symmetry breaking and the effective softening of the constraint lead to excitations described by the spin $\frac{1}{2}$ two–component complex field z_α, with massive dispersion relations. However, the self–generated gauge field $A_\mu = z^*_\alpha\partial_\mu z_\alpha$ confines the spin $\frac{1}{2}$ degrees of freedom and the actual excitations are described by a composite spin 1 spin–wave field. As we shall see, the coupling to holons will induce deconfinement

of spin $\frac{1}{2}$ excitations. In terms of fields b_α, (slave fermion approach), one realises that the $z-$field in the reduced Brillouin zone with two complex components corresponds (at $\delta = 0$) to appearance of an $s + id$ RVB order parameter with a vanishing gap, in MFA, at four points $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$ [22]. The NLσ model for spinons can also be derived in this representation. The low–energy excitations of this model are fluctuations around these points turned into massive ones by the $\langle \bar{V}_z^2 \rangle$ term. Physically, this is a coexisting π flux plus $s + id$ RVB state. From our estimate we expect the RVB $s+id$ order parameter (4) to be much smaller then the AM order parameter (3). These features are clearly shown in the numerical MFA calculations of [23].

Now turn to holons. We use a $U(1)$ gauge with $e^{i\int_{\langle ij \rangle} \bar{B}}$ being purely imaginary and assume spinons are in disordered phase with AM parameter ~ 1. In the rescaled ε lattice, neglecting higher order terms in ε and b, the effective action describes the usual 2–component Dirac ("staggered") fermions of the flux phase [24], with vertices of the double–cone dispersion relations in the reduced Brillouin zone centered at $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$ (in the $\varepsilon = 1$ lattice), with chemical potential δ.

Define the four sublattices: (1) for j_1, j_2 even, (2) for j_1 odd j_2 even, (3) for j_1 even j_2 odd, (4) for j_1, j_2 odd; they can be grouped into two "Nèel sublattices" $A = \{(1), (4)\}, B = \{(2), (3)\}$. The holon field restricted to the sublattice $(\#)$ is denoted by $H^{(\#)}$. Set:

$$\Psi^{(1)} \equiv \begin{pmatrix} \Psi_A^{(1)} \\ \Psi_B^{(1)} \end{pmatrix} \equiv \begin{pmatrix} e^{-i\frac{\pi}{4}} H^{(1)} + e^{i\frac{\pi}{4}} H^{(4)} \\ e^{-i\frac{\pi}{4}} H^{(3)} + e^{i\frac{\pi}{4}} H^{(2)} \end{pmatrix}$$
$$\Psi^{(2)} \equiv \begin{pmatrix} \Psi_B^{(2)} \\ \Psi_A^{(2)} \end{pmatrix} \equiv \begin{pmatrix} e^{-i\frac{\pi}{4}} H^{(2)} + e^{i\frac{\pi}{4}} H^{(3)} \\ e^{-i\frac{\pi}{4}} H^{(4)} + e^{i\frac{\pi}{4}} H^{(1))} \end{pmatrix}$$
$$\gamma_0 = \sigma_z, \gamma_\mu = (\sigma_y, \sigma_x), \not{A} \equiv \gamma_\mu A^\mu, \not{\partial} \equiv \gamma_\mu \partial_\mu, \bar{\Psi}^{(\#)} = \gamma^0 \Psi^{(\#)\dagger}$$

and assign charge $e_A = +1 (e_B = -1)$ to the fields on the $A(B)$ sublattice. Then, neglecting short range interactions, we obtain the low–energy continuum action for holons:

$$S_h^\star = \int d^3x \sum_{r=1}^{2} \bar{\Psi}^{(r)} \Big(\gamma_0 (\partial_0 - \delta - e^{(r)} A_0) + t(\not{\partial} - e^{(r)} \not{A}) \Big) \Psi^{(r)}$$

Here A_μ is nothing but the gauge field for the h/s gauge. We can use $S^\star = S_s^\star + S_h^\star$ to compute, as in [25], the gauge field propagator induced by the spinon and holon vacuum polarisation. Since the spinon is massive, the corresponding vacuum polarization would be Maxwell–like. Hence, in the absence of holons, it would logarithmically confine the spinons. However, excitations represented by $\Psi_B^{(1)}$ and $\Psi_A^{(2)}$ describe a FL with a small FS ($\varepsilon_F \sim O(\delta t)$) in the reduced Brillouin zone around the points $(\frac{\pi}{2}, \pm\frac{\pi}{2})$. Thus the vacuum polarization exhibits the Reizer singularity [5] and the full gauge interaction is <u>not</u> confining. Nevertheless, since we are in 2D, the attractive force mediated by the gauge field is expected to produce bound states neutral w.r.t. the h/s gauge, i.e. bound states with quantum numbers of the spin wave (for a rigorous discussion of a similar problem, see [26]) and the electron [27], respectively. For this reason, neglecting "semion" nature can be justified to some extent in 2D. Even if we neglect the gauge fluctuations, the existence of two bands in the reduced Brillouin zone gives rise to a "shadow band" effect (the spectral weight for the part of the pocket facing (π, π) is greatly reduced) due to the presence of $\gamma-$matrices, leading to mixing of fermions of these bands. The situation is similar to the slave boson case [13].

To conclude we summarize the main differences between 1D and 2D cases. In $1D$: the NLσ model of spinons contains a $\theta = \pi$ topological term, yielding deconfinement; absence of $\bar{V}$ term makes the spinons massless; there is no $\bar{B}$ term and there is only one holon band; the h/s gauge field A vanishes, hence there is no attractive gauge force between holons and spinons, so their statistics appears to be crucial. In contrast, in 2D, without taking into account the "semion" nature of holons and spinons, but considering the feedback of holons on the $SU(2)$ gauge field, producing the spinon gap, we can already understand quite a number of peculiar properties for underdoped cuprates: normal state pseudogap, small FS, shadow bands, etc. The main features of the MF calculation [13] survive gauge fluctuations. Further consideration of these fluctuations between holons and spinons provides a binding force between them, and this composite electron shows non-FL behavior, like linear T dependence of resisitivity [14], and others. More detailed consideration of various physical properties within the present model will be given elsewhere [18].

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REFERENCES

[1] B.I. Shraiman and E.D. Siggia, Phys. Rev. Lett. **61**, 467 (1988), **62**, 1564 (1989)

[2] See, e.g, A.J. Millis, *et. al.*, J. Phys. Chem. Solids **56**, 1641 (1995)
130 (1994).

[3] A.G. Loeser, *et al.*, Science **273**, 325 (1996); H. Ding, *et al.*, Nature (London) **382**, 51 (1996).

[4] P.W. Anderson, Science **235**, 1196 (1987)

[5] P.A. Lee and N. Nagaosa, Phys. Rev. B. **46**, 5621 (1992)

[6] X.G. Wen and P.A. Lee, Phys. Rev. Lett. **76**, 503 (1996);
P.A.Lee, N. Nagaosa, T.K. Ng, and X.G. Wen, Preprint cond-mat/9701168.

[7] A.P. Kampf and J.R. Schrieffer, Phys. Rev. B **41**, 6399 (1990).

[8] A.V. Chubukov, D.K. Morr, and K.A. Shakhnovich, Phil. Mag. B **74**, 563 (1996); A. V. Chubukov and D.K. Morr, Preprint cond-mat/9701196

[9] J. Fröhlich, T. Kerler, P.A. Marchetti, Nucl. Phys. B **374**, 511 (1992); J. Fröhlich, P.A. Marchetti, Phys. Rev. B **46**, 6535 (1992).

[10] R.B. Laughlin, Science **242**, 525 (1988)

[11] P. A. Marchetti, Zhao–Bin Su, Lu Yu, Nucl. Phys. B **482** [FS], 731 (1996).

[12] J. Affleck and J.B. Marston, Phys. Rev. B **37**, 3774 (1988)

[13] Xi Dai, Zhao–Bin Su, Lu Yu, Phys. Rev. B, 1 August, 1997 [BB6481].

[14] L. B. Ioffe and P.B. Wiegmann, Phys. Rev. Lett. **65**, 653 (1990). The $U(1)$ effective action derived in this paper looks similar to the one considered there, but those authors considered holes in the Nagaoka state, while we start from the AF state.

[15] See, e.g., Lu Yu, Zhao-Bin Su, and Yanmin Li, Chin. J. Phys. (Taipei) **31**, 579 (1993)

[16] J. Fröhlich, D. Brydges, E. Seiler, Ann. Phys. **121**, 227 (1979).

[17] Y. Hagesawa, P. Lederer, T.M. Rice, P.B. Wiegmann, Phys. Rev. Lett. **63**, 907 (1989); J. Bellissard, R. Rammal, Europhys. Lett. **13**, 205 (1990)

[18] P.A. Marchetti, Zhao–Bin Su, Lu Yu, in preparation.

[19] E. Fradkin, "Field Theories of Condensed Matter Systems" (Addison–Wesley, New York 1991).

[20] S.F. Edwards, A. Lenard, J. Math. Phys. **3**, 778 (1962); J.M. Kosterlitz, D.V. Thouless, J. Phys. C **6**, 1181 (1973).

[21] D.F. Escande and B. Souillard, Phys. Rev. Lett. **52**, 1296 (1984)

[22] D. Yoshioka, J. Phys. Soc. Jpn. **58**, 32, 1516 (1989)

[23] D.N. Sheng, Zhao–Bin Su, Lu Yu, Phys. Rev. B **42**, 8732 (1990).

[24] The Dirac form for the holon action in the presence of π flux was considered earlier by several authors (see, *e.g.*, N. Dorey and N.E. Mavromatos, Phys. Rev. B **44**, 5288 (1991)). However, the presence of $\gamma_0\delta$ term in our action S_h^* gives rise to Reizer singularity,leading to different physics.

[25] L. Ioffe and A. Larkin, Phys. Rev. B **39**, 8988 (1989).

[26] J. Fröhlich, J. Bricmont, Nucl. Phys. B **280** [FS], 385 (1987)

[27] Y. Chen, D. Förster, and A. Larkin, Phys. Rev. B **46**, 5370 (1992); See, also, P. Beran, D. Poilblanc, and R.B. Laughlin, Preprint cond-mat/9505085